GEOPHYSICAL RESEARCH LETTERS, VOL. 36, L11818, doi:10.1029/2008GL037111, 2009

Do all raindrops fall at terminal speed?

Guillermo Montero-Martínez,[1,2] Alexander B. Kostinski,[3] Raymond A. Shaw,[3]
and Fernando García-García[1,2]

Received 22 December 2008; revised 18 April 2009; accepted 11 May 2009; published 13 June 2009.

[1] A unique relation between raindrop size and fall speed $v_t(D)$ is assumed throughout atmospheric science. Yet, our speed versus size measurements of millions of drops during natural rainfall events show that many intermediate sized raindrops fall up to an order of magnitude faster than expected. Furthermore, images of drop clusters reveal that these "super-terminal drops" are differently sized fragments of a recent break-up, moving with the speed of the parent drop and relaxing towards $v_t(D)$. Additional evidence of the break-up conjecture includes: (i) positive skewness in the distribution of fall speed deviations, (ii) strong size dependence of fall speed deviations and their maximum values and, (iii) preponderance of super-terminal drops in the presence of large raindrops (i.e., during periods of high rainfall rates). **Citation:** Montero-Martínez, G., A. B. Kostinski, R. A. Shaw, and F. García-García (2009), Do all raindrops fall at terminal speed?, *Geophys. Res. Lett.*, *36*, L11818, doi:10.1029/2008GL037111.

1. Introduction

[2] Raindrops come in different sizes: from a few hundred micrometers to several millimeters. As it is well known, larger drops fall faster than smaller ones and there is a thoroughly studied and experimentally tested one-to-one correspondence between the drop size and its terminal speed [*Gunn and Kinzer*, 1949; *Beard and Pruppacher*, 1969; *Beard*, 1976; *Hosking and Stow*, 1991; *Testik and Barros*, 2007], the latter resulting from a balance between gravity and air resistance. The $v_t(D)$ relation is closely tied to the notion of the drop size distribution (hereafter, DSD), a fundamental aspect of Doppler radar meteorology and precipitation measurement science, and therefore underlying many hydrological applications [*Doviak and Zrnić*, 1993; *Colier*, 1996; *Salles and Creutin*, 2003]. Indeed, weather radar has become an indispensable part of modern public service [*Colier*, 1996] and most of its quantitative uses in meteorology and hydrology rely on daily and even hourly measurements of the rainfall rate, R. Furthermore, R is defined in terms of $v_t(D)$ and has the dimensions of speed ($R \propto \int D^3\ v_t(D)\ dD$, typically measured in mm/hr). In spite of the importance of the $v_t = v_t(D)$ relation, most experiments have been conducted in either a laboratory setting or did not have coincident size and speed information. Despite the fact that both have been measured with consistent reports of predominantly super-terminal fall speeds, these deviations were variously attributed to instrumental errors that include splashing and sampling conditions (such as the presence of updrafts or turbulence) or dismissed altogether as outliers [*Donnadieu*, 1980; *Hauser et al.*, 1984; *Hosking and Stow*, 1991; *Kruger and Krajewski*, 2002]. So, it is argued here that the statement that raindrops fall at terminal speed must be regarded as an assumption. Hence, the purpose of this work is to report on field data tests of this assumption with surprising results. In what follows, the theoretical motivation for questioning the assumption is outlined.

[3] DSDs of natural rainfall are broad and often exponential [*Marshall and Palmer*, 1948], with large drops being relatively rare. Because of the terminal speed dependence on size, large drops catch up with smaller ones and occasional coalescence occurs. It is impossible, in such a framework, to have drops of vastly different sizes moving at the same speed. However, large drops eventually break up, either because they become hydrodynamically unstable or as a result of temporary coalescence produced after a collision [*Pruppacher and Klett*, 1997]. Immediately thereafter, the pieces, disembarking off a common carrier, move at about the same high speed. Since it has long been realized that interplay of break-up and coalescence determines the evolution of the DSD in natural rain [*Langmuir*, 1948], an important question arises: does break-up or coalescence affect the fall speeds of raindrops and, if so, how? Once asked, the question is readily answered. The result of a coalescence is a single drop falling nearly at the same speed as the larger of the two coalescing drops and, therefore, slightly slower than its terminal speed. The break-up results in several fragments [*Testik and Barros*, 2007], all moving at the same speed, and, therefore, the smallest of the fragments moving much faster than its terminal speed. For example, a 100 μm fragment is expected to break off at the speed of a parent drop (say, 4 mm in diameter) which is nearly two orders of magnitude faster than the corresponding terminal speed. In fact, strong evidence for drop breakup causing super-terminal speed would be an observation of drops of vastly different sizes in physical proximity, all moving at the same speed. A convenient quantity to be defined at this point is the ratio $v(t, D)/v_t(D)$, where $v(t, D)$ is the observed speed. The time-dependence is included to keep in mind that, in addition to size, the *observed* speed depends on the time elapsed since the break-up as the fragment relaxes towards its $v_t(D)$.

2. Data and Methods

[4] Microphysical data were obtained during natural rainfall events at the Mexico City campus of the National University of Mexico during 2002, 2004 and 2006. Here we

---

[1] Posgrado en Ciencias de la Tierra, Universidad Nacional Autónoma de México, Circuito de la Investigación Científica, Mexico City, Mexico.

[2] Centro de Ciencias de la Atmósfera, Universidad Nacional Autónoma de México, Circuito de la Investigación Científica, Mexico City, Mexico.

[3] Department of Physics, Michigan Technological University, Houghton, Michigan, USA.

0094-8276/09/2008GL037111$05.00

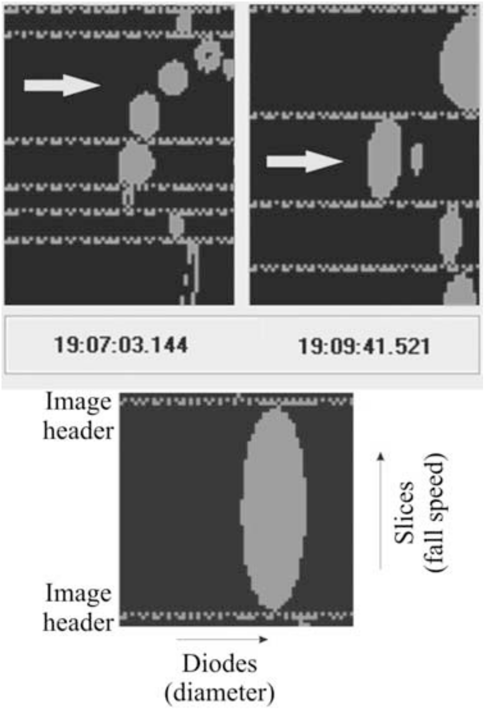

Figure 1. Super-terminal break-up fragments caught by the 2DC with a sampling frequency of 333.3 kHz. (bottom) Two-dimensional images form as drops fall past a linear diode array, so that horizontal dimension gives drop diameter and vertical dimension gives drop speed. Detection events are separated by horizontal bar codes – the image header. (top left) A single detection event is identified: four drops, three of them with D = 0.250 mm and one with D = 0.125 mm, having similar fall speeds ranging from $6.2 < v < 8.1$ m s^{-1} but all with $[v/v_t\,(D)] > 6$. This is consistent with the expected motion of fragments shortly after a break-up. (top right) Another event is shown: two drops (D = 0.274 and 0.096 mm) moving at nearly the same speed (v = 4.0 and 3.6 m s^{-1}, respectively). Note that the smaller drop is falling slightly slower, consistent with faster relaxation to terminal speed for smaller drops; this in spite of the fact that $(v/v_t) = 3.6$ for the large drop and $(v/v_t) = 14.9$ for the small drop, consistent with the fragmentation hypothesis.

present results based on approximately 64,000 drops with diameters between 0.1 and 3 mm. Raindrop size and fall speed data were gathered using two optical array spectrometer probes [*Knollenberg*, 1981] fixed at the ground in a vertical orientation. The drop sizing nominal ranges for the 2DC and 2DP devices are 20 to 800-, and 200 to 6400-μm, respectively. Drop images were obtained using Particle Analysis and Collection System (PACS) commercial sampling software [*Droplet Measurement Technologies*, 2001], although detailed analyses were done with a reconstruction algorithm based on *Heymsfield and Parrish* [1978] and developed specially for this sampling technique [*Álvarez-Pimentel and Torreblanca-Beltrán*, 1992]. The drop size is determined from the maximum width across the array, whereas its fall speed is calculated by dividing the minor axis, corresponding to the drop shape deformation [*Green*, 1975], by the number of time-slices and the sampling frequency of the probe. Large values of sampling frequency allow one to improve the resolution for drop fall speed, depending on the calibration of the instruments. Usually, the sampling frequency for the 2DC (333.3 kHz in the case of images in Figure 1) is larger than that for the 2DP. Distributions taken at high rain rate simultaneously with both instruments are mutually consistent, especially considering that the 2DC probe, having a smaller detection cross-section, samples about 50 times fewer drops than the 2DP spectrometer. In order to rule out artifacts due to drop splashing on instruments - which has been a standard explanation for the handful of prior observations of raindrops with $v/v_t \gg 1$ from optical disdrometers [*Hosking and Stow*, 1991; *Donnadieu*, 1980; *Hauser et al.*, 1984; *Kruger and Krajewski*, 2002] – the data presented here were restricted to calm conditions, i.e., average horizontal wind speed of only 0.6 m/s and bounded by 2 m/s.

3. Results

[5] The evidence for the above-mentioned fast-moving clusters is presented in Figure 1, where two examples of groups of "super-terminal drops" are shown: first, a cluster of four super-terminal drops with similar fall speeds; and second, a cluster of two super-terminal drops of vastly different sizes but, again, both moving at essentially the same speed. The odds of encountering such events with uncorrelated drops are vanishingly small; yet, both groupings are consistent with the motion of fragments shortly after a break-up. Usually, multi-drop images are not inspected by automated data processing algorithms, but we searched for them during post-processing motivated by the arguments here presented.

[6] Although striking, the evidence just presented for such fast-moving clusters does not necessarily tell one whether breakups are frequent enough to cause significant deviations from the terminal speed distribution. To that end, we examine the statistical significance of our data in terms of the break-up conjecture.

[7] Our first and, perhaps, most striking prediction is the dependence of $v(t, D)/v_t(D)$ on drop size. Indeed, $v_t(D) \propto D$ in the drizzle range [*Testik and Barros*, 2007; *Gunn and Kinzer*, 1949; *Hosking and Stow*, 1991; *Beard and Pruppacher*, 1969; *Beard*, 1976; *Colier*, 1996], so assuming the maximum of $v(t, D)$ as the speed of the parent drop results in $v(t, D)/v_t(D) \approx D_{parent}/D_{fragment}$. In addition, large drops resulting from coalescence are expected to fall slightly slower than their terminal fall speed, but attain the latter within some characteristic relaxation time [*Wang and Pruppacher*, 1977]. The opposite is expected for the breakup fragments which begin to move fast but relax to terminal speed. As fragments are more numerous, a substantial asymmetry in the distribution of $v(t, D)/v_t(D)$ is expected: small negative deviations and large positive deviations. The importance of the effect depends on the ubiquity of breakups and on the ratio of relaxation time to inter-collision time: as R increases, so does the number of large drops and, therefore, the number of fragmentation events. Hence, another anticipated effect is more skewness in the $v(t, D)/v_t(D)$ distribution as R increases.

[8] In Figure 2, the overall significance of the break-up scenario is demonstrated. The characteristic effects on (v/v_t) are borne out in Figure 2a, which illustrates the average observed drop speed ratio, $(\overline{v}/v_t)$, for three different rain rates.

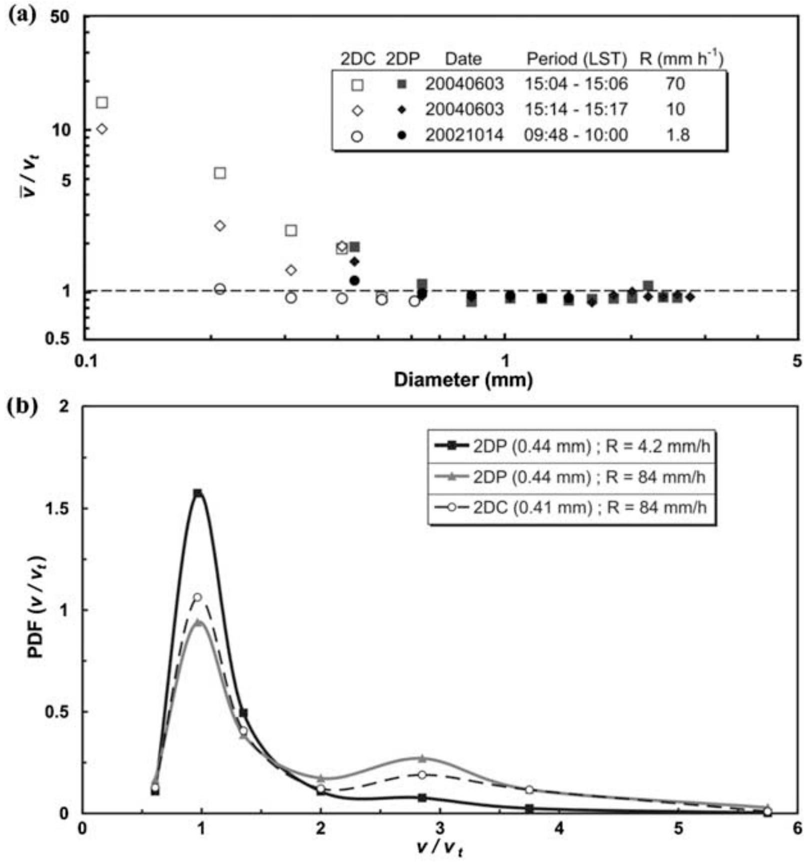

Figure 2. Evidence for the statistical significance of the break-up scenario. (a) Size dependence of the speed anomaly, $(\overline{v}/v_t)$, averaged over hundreds of events per point, for three rain rates. Size range of about 0.1 to 3 mm is attained with two independent optical devices with different size range resolution. Despite different size resolutions and sampling frequencies, there is agreement within the overlapping range on the fraction as well as on the speed values of super-terminal drops. Inspection of weakest rainfall ($R = 1.8$ mm h^{-1}, no drops larger than 1.5 mm) reveals $(v/v_t) \approx 1$ over the whole drop size range. (b) Estimated probability densities of (v/v_t) for $D \approx 0.400$ mm, size detected by both instruments. The rain rates are weighted (by number of sampled drops) averages. The number of drops sampled by the 2DP for $R = 84$ and 4.2 mm hr^{-1} are $N = 13{,}186$ and 2353, respectively; and $N = 123$ for the 2DC (due to its smaller cross sampling area). The possibility that instrumental artifacts would conspire to produce such patterns in both instruments and with respect to size, rain rate, and asymmetry, appears remote.

On the other hand, Figure 2b shows estimated probability density functions for (v/v_t) for a 400-μm drop, chosen because both instruments can observe it, thereby providing an additional consistency check. Note that the distributions are highly skewed, with positive tails of the high rain rate distributions extending to $(v/v_t) \approx 5$, and the skewness increases with R, as anticipated above.

[9] As seen in Figure 2b, up to 50% of 0.44-mm drops are super-terminal during intense ($R > 50$ mm/hr) events. (Super-terminal and sub-terminal are defined here as $(v/v_t) \geq 1.3$ and $(v/v_t) \leq 0.7$, respectively, based on instrument uncertainty of approximately 30%.) As rainfall rate decreases, so does the number of super-terminal drops. Thus, the fraction of 0.44-mm super-terminal drops decreases to less than 20% for $R = 10$ mm/hr. Furthermore, super-terminal fractions vary with size. For sizes not shown in Figure 2b, e.g., for $D \approx 0.24$ mm, super-terminal fractions are 80% during heavy rain periods, decreasing to 15% for periods of low rainfall rate, and for $D \approx 0.64$ mm drops super-terminal fractions are 20% for $R > 60$ mm/h, decreasing to 1–2% for $R < 10$ mm/h. In contrast, sub-terminal fractions are less than 5% (regardless of drop size) with deviations within the instrumental uncertainty.

4. Discussion

[10] The observed increase in the fraction of super-terminal drops (e.g., $(v/v_t) \geq 1.3$) with rain rate has implications for our understanding of the physics of rain. The relative importance of the collision-breakup-relaxation process can be described by the agitation parameter $\alpha \equiv \tau_t/\tau_c$; the ratio of the drop speed relaxation time (τ_t) to the mean inter-collision time (τ_c). Rain with $\alpha \gg 1$ is then highly agitated, with collisions preventing drops from attaining terminal speeds. Existing rain microphysics models are based on the implicit assumption that $\alpha \ll 1$, with all drops falling at $v_t(D)$, but our data suggest that this is not the case. Estimates confirm that at high rain rates the agitation parameter can approach unity: raindrops in the size range considered here require about a second to reach terminal speed [*Wang and Pruppacher*, 1977] and the mean time between collisions for realistic DSDs is comparable

[*McFarquhar and List*, 1991]. Moreover, our data are consistent with these agitation parameter estimates, particularly when skewness of the free path distribution is taken into account. Skewness in the (v/v_t) distribution also has important consequences for the evolution of DSDs, thought to result from interplay of coalescence growth and drop break-up [*Low and List*, 1982; *Barros et al.*, 2008]. Central to the latter process is the collision kinetic energy [*Low and List*, 1982]: $K_c \propto [(D_i^3 D_j^3)/(D_i^3 + D_j^3)][v_t(D_i) - v_t(D_j)]^2$, where subscripts i and j refer to different drop size categories. However, as α approaches unity, this must be changed to $K_c \propto (v_i - v_j)^2$, where v_i and v_j are now regarded as random variables, whose distributions posses pronounced tails as in Figure 2b.

[11] Precipitation is recognized as one of the poorly quantified aspects of the hydrological cycle [*Chahine*, 1992]. As the ability to measure and/or calculate precipitation rates from first principles in climate models has enormous societal implications, even incremental improvements in measurement and prediction abilities can translate to significant economic benefits [*Freebairn and Zillman*, 2002]. This study concludes with just two examples, illustrating practical consequences of the observed raindrop speed distributions: ground-based and radar measurements of raindrop size distributions and rain rates. The celebrated Joss-Waldvogel disdrometer has proven to be an exceptionally robust and widely used instrument for decades [*Joss and Waldvogel*, 1967], assigning raindrop size by associated impact momentum, via the assumption that all drops travel at $v_t(D)$. The skewness in the v/v_t distribution reported here (Figure 2), produces spuriously large drops, progressively so with increasing rain rate. Modern video disdrometers can avoid such problems because raindrop diameter and fall speed are independently measured, but often drops falling at speeds deviating as little as ±40% from the terminal speed are filtered as outliers [*Hosking and Stow*, 1991; *Hauser et al.*, 1984].

[12] Rain and drizzle DSDs are commonly retrieved from Doppler spectra measured by upward pointing radars [*Sheppard and Joe*, 2008]. While such inversions of the raindrop size distribution from the measured Doppler spectrum all depend on the terminal fall speed assumption [*Sekhon and Srivastava*, 1971; *Frisch et al.*, 1995], they may be insensitive to super-terminal speed of the smaller drops because of the D^6 factor in radar reflectivity. However, the spuriously large drops inferred from disdrometers, by making the phantom contribution to rainfall rate, can possibly bias *Z-R* relations, when compared to the radar measurements [*Salles and Creutin*, 2003].

[13] **Acknowledgments.** The authors are indebted to Jorge Escalante, Wilfrido Gutiérrez and Alfredo Rodríguez for their continuous technical support. This work was supported in part by the US National Science Foundation through grants ATM05-54670 and ATM05-35488, and by the Coordinación de la Investigación Científica-UNAM through a grant from its *Programa de Intercambio Académico Internacional*.

References

Álvarez-Pimentel, J. M., and J. Torreblanca-Beltrán (1992), Desarrollo de un sistema de software para interpretación y análisis de datos de espectrómetros de gotas, B.Sc. thesis, 150 pp., Univ. Nac. Auton. de Mex., Mexico City.

Barros, A. P., O. P. Prat, P. F. Shrestha, Y. Testik, and L. F. Bliven (2008), Revisiting Low and List (1982): Evaluation of raindrop collision parameterizations using laboratory observations and modeling, *J. Atmos. Sci.*, *65*, 2983–2993.

Beard, K. V. (1976), Terminal velocity and shape of cloud and precipitation drops aloft, *J. Atmos. Sci.*, *33*, 851–864.

Beard, K. V., and H. R. Pruppacher (1969), A determination of the terminal velocity and drag of small water drops by means of a wind tunnel, *J. Atmos. Sci.*, *26*, 1066–1072.

Chahine, M. T. (1992), The hydrological cycle and its influence on climate, *Nature*, *359*, 373–380.

Colier, C. G. (1996), *Applications of Weather Radar Systems: A Guide to Uses of radar Data in Meteorology and Hydrology*, 390 pp., John Wiley, New York.

Donnadieu, G. (1980), Comparison of results obtained with the VIDIAZ spectropluviometer and the Joss-Waldvogel rainfall disdrometer in a "rain of a thundery type", *J. Appl. Meteorol.*, *19*, 593–597.

Droplet Measurement Technologies (2001), *Particle Analysis and Collection Software* (*PACS*), *Version 2.2, Users Manual* [CD-ROM], Droplet Meas. Technol., Boulder, Colo.

Doviak, R. J., and D. S. Zrnić (1993), *Doppler Radar and Weather Observations*, 2nd ed., 562 pp., Academic, Mineola, N. Y.

Freebairn, J. W., and J. W. Zillman (2002), Economic benefits of meteorological services, *Meteorol. Appl.*, *9*, 33–44.

Frisch, A. S., C. W. Fairall, and J. B. Snider (1995), Measurement of stratus cloud and drizzle parameters in ASTEX with a K_α-Band Doppler radar and a microwave radiometer, *J. Atmos. Oceanic Technol.*, *52*, 2788–2799.

Green, A. W. (1975), An approximation to the shapes of large raindrops, *J. Appl. Meteorol.*, *14*, 1578–1583.

Gunn, R., and G. R. Kinzer (1949), The terminal velocity of fall for water drops in stagnant air, *J. Meteorol.*, *6*, 243–248.

Hauser, D., P. Amayenc, B. Nutten, and P. Waldteufel (1984), A new optical instrument for simultaneous measurement of raindrop diameter and fall-speed distribution, *J. Atmos. Oceanic Technol.*, *1*, 256–269.

Heymsfield, A. J., and J. L. Parrish (1978), A computational technique for increasing the effective sampling volume of the PMS two-dimensional particle size spectrometer, *J. Appl. Meteorol.*, *17*, 1566–1572.

Hosking, J. G., and C. D. Stow (1991), Ground-based measurements of raindrop fallspeeds, *J. Atmos. Oceanic Technol.*, *8*, 137–147.

Joss, J., and A. Waldvogel (1967), A raindrop spectrograph with automatic analysis, *Pure Appl. Geophys.*, *68*, 240–246.

Knollenberg, R. G. (1981), Techniques for probing cloud microstructure, in *Clouds: Their Formation, Optical Properties, and Effects*, edited by P. V. Hobbs and A. Deepak, pp. 15–89, Academic, New York.

Kruger, A., and W. F. Krajewski (2002), Two-dimensional video disdrometer: A description, *J. Atmos. Oceanic Technol.*, *19*, 602–617.

Langmuir, I. (1948), The production of rain by a chain reaction in cumulus clouds at temperatures above freezing, *J. Meteorol.*, *5*, 175.

Low, T. B., and R. List (1982), Collision, coalescence and breakup of raindrops. Part I: Experimentally established coalescence efficiencies and fragment size distributions in breakup, *J. Atmos. Sci.*, *39*, 1591–1606.

Marshall, J. S., and W. M. Palmer (1948), The distribution of raindrops with size, *J. Meteorol.*, *5*, 165–166.

McFarquhar, G. M., and R. List (1991), The raindrop mean free path and collision rate dependence on rainrate for three-peak equilibrium and Marshall-Palmer distributions, *J. Atmos. Sci.*, *48*, 1999–2003.

Pruppacher, H. R., and J. D. Klett (1997), *Microphysics of Clouds and Precipitation*, 2nd ed., 954 pp., Kluwer Acad., Dordrecht, Netherlands.

Salles, C., and J. Creutin (2003), Instrumental uncertainties in Z-R relationships and raindrop fall velocities, *J. Appl. Meteorol.*, *42*, 279–290.

Sekhon, R. S., and R. C. Srivastava (1971), Doppler radar observations of drop-size distributions in a thunderstorm, *J. Atmos. Sci.*, *28*, 983–994.

Sheppard, B. E., and P. I. Joe (2008), Performance of the precipitation occurrence sensor system as a precipitation gauge, *J. Atmos. Oceanic Technol.*, *25*, 196–212.

Testik, F. Y., and A. P. Barros (2007), Toward elucidating the microstructure of warm rainfall: A survey, *Rev. Geophys.*, *45*, RG2003, doi:10.1029/2005RG000182.

Wang, P. K., and H. R. Pruppacher (1977), Acceleration to terminal velocity of cloud and raindrops, *J. Atmos. Sci.*, *16*, 275–280.

F. García-García, Centro de Ciencias de la Atmósfera, Universidad Nacional Autónoma de México, Circuito de la Investigación Científica, Ciudad Universitaria, 04510 México, D. F., Mexico. (ffgg@unam.mx)

A. B. Kostinski and R. A. Shaw, Department of Physics, Michigan Technological University, 1400 Townsend Drive, Houghton, MI 49931, USA.

G. Montero-Martínez, Posgrado en Ciencias de la Tierra, Universidad Nacional Autónoma de México, Circuito de la Investigación Científica, Ciudad Universitaria, 04510 México, D. F., Mexico.